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METHOD OF MODELLING GAS TURBINE PERFORMANCE

The basis of a combined cycle gas turbine mass and energy balance, carried at the concept stage, preliminary design or during analysis, is the calculation of the gas turbine design point. This article presents a gas turbine unit mathematical modelling process intended for implementation of the model in a computer program. The aim of the development was a relatively simple, practical from engineer's point of view, and universal gas turbine model with a classic, open system design: compressor – combustion chamber – turbine. Attempts have been made to maintain the accuracy of modelling and to keep the ability for extension of the model so to be able to simulate the operation of the turbine at off-design conditions, in particular at part load. Stationary operation of the unit at nominal load is assumed. Thermodynamic system modelling methods, on the level of its global structure, have been submitted. Problems related to the aforementioned issues are discussed and ways to resolve them proposed. The developed model has been implemented in a dynamic link library written in C++ language with usage of C/C++ Minpack numerical library [Keating 2007, Razak 2007].

Key words**:** gas turbine, Brayton cycle, numerical simulation, mathematical modelling, steady state analysis, energy system

1. INTRODUCTION

One of the ways to increase efficiency of fuel chemical energy conversion to mechanical work (that may be further converted into electrical energy), and/or heat in a convenient form, e.g. contained in steam, is interfacing two thermodynamic

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systems, assuming that the waste heat of either constitutes the upper heat source of the other. Broad approval for the combination of a gas turbine unit (gas turbine in common parlance) and a steam cycle has been achieved mainly due to its high efficiency and technological maturity. In a combined cycle, a gas turbine exhaust heat contained in a flue gas is recovered in a steam generator, and then converted into a useful form of energy, thus increasing the efficiency of the whole system.

Creating a conceptual design, or analysing an existing CCGT, we should start by creating a mathematical model of a gas turbine unit, operating in its nominal conditions. Such a proceeding will afford us to obtain basic operational parameters of the gas turbine, including essential input data for the steam cycle calculations.

Fig. 1 presents the established structural model of a gas turbine. The arrows on the shaft of the gas turbine unit represent turbine power distribution. The element of the model, marked as ΔT, has no reflection in reality (does not exist physically), and means a drop of combustion gas temperature after the combustion chamber. The temperature of the combustion gas flow going to the first turbine stage is lower than the mean combustion temperature. Additionally, the ΔT element partially compensates for the lack of a turbine cooling model, which is omitted because to model it precisely, the design details of the cooling system of the particular gas turbine unit would have to be known – most specifically: the type of cooling, the number of cooled guide and rotor vanes, the highest turbine blade surface temperature and the cooling air bleed points. The problem is the quite sizable error of the mass flow rate of air after the compressor. In reality, the air mass flow rate after the compressor is the difference between the air mass flow being sucked on its inlet and a sum of air mass streams being extracted from the compressor for turbine cooling purposes. Overestimated air mass flow rate after the compressor, results in overestimated compressor power consumption and thus, underestimated efficiency of the gas turbine unit. To compensate this, compressor polytropic efficiency may be increased (against real/measured). Owing to the insertion of the ΔT element it is possible to balance the whole system while keeping real or close to real turbine polytropic efficiency.

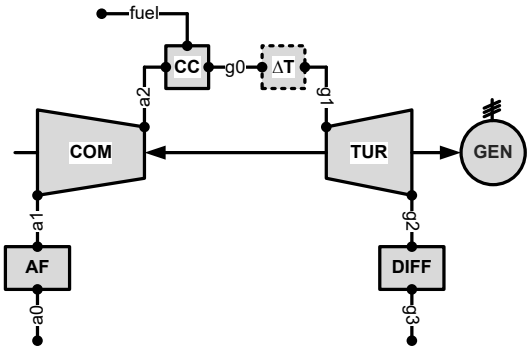

Fig. 1. Model of a gas turbine unit

Essential processes that occur in a typical gas turbine unit are shown in Fig. 2. Working media undergo the following processes:

a0–a1 intake/suction of ambient air;
throttling of the air flow in the filter (close to isenthalpic),
a1–a2 compression of air in the compressor,
a2–g1 combustion of fuel in the combustion chamber (usually at several percentage pressure loss), in the presence of air fed by the compressor,
g1–g2 expansion of combustion gas in the turbine,
g2–g3 throttling of the exhaust gas flow (close to isenthalpic);
discharge of exhaust gas to the environment.

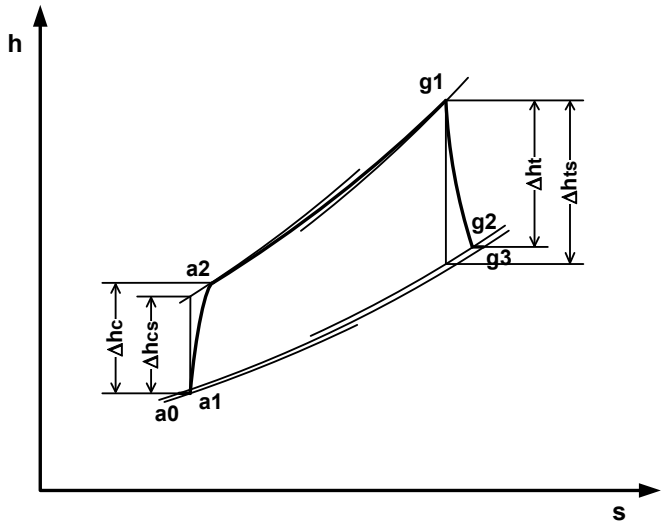

Fig. 2. Gas turbine ‘cycle’, enthalpy–entropy diagram

2. ASSUMPTIONS

Available catalogue data of gas turbine units include the following entities: frequency, electric power, electric efficiency, compressor pressure ratio, temperature after diffuser and exhaust gas mass flow rate. These parameters are insufficient to carry out a simulation, therefore the values of missing parameters have to be estimated. A list of necessary parameters used as an input data in a developed model of gas turbine unit, with missed values estimation difficulty, are contained in table no 1.

Table 1. Input data

Parameter		Known / Unknown	Difficulty of estimation
N_{el}	electric power	known	n/a
η_{el}	electric efficiency	known	n/a
π	compressor pressure ratio	known	n/a
T_{g3}	exhaust gas temperature	known	n/a
m_{g3}	exhaust gas mass flow rate	known	n/a
η_{mech}	mechanical efficiency of the gas turbine unit	unknown	•
η_{GEN}	generator electric efficiency	unknown	•
η_{CC}	combustion chamber efficiency	unknown	•
Δp_{AF}	air pressure drop in the filter	unknown	•
Δp_{DIFF}	exhaust gas pressure drop at the diffuser	unknown	•
ΔT_{DIFF}	exhaust gas temperature drop at the diffuser	unknown	•
ξ_{CC}	relative pressure loss in combustion chamber	unknown	••
ΔT_{CC}	difference between the mean combustion temperature and combustion gas temperature before the turbine	unknown	••
T_{g0}	mean combustion temperature	unknown	•••
$\eta_{p,COM}$	compressor polytropic efficiency	unknown	•••
$\eta_{p,TUR}$	turbine polytropic efficiency	unknown	•••
LHV	fuel lower heating value; fuel is assumed to be 100% methane; methane LHV = 50016 kJ/kg	known	n/a
T_{fuel}	fuel temperature; assumed to be ISO temperature (15°C)	known	n/a
p_{atm}, T_{atm}	atmospheric pressure, temperature, and relative humidity; assumed to be ISO conditions (1,013 bar, 15°C, 60%)	known	n/a

Values of the pressure ratio and exhaust gas mass flow rate are to be taken from the catalogue, whereas electric power, electric efficiency and exhaust gas temperature after diffuser are calculated values. After the initial assumption of unknown input parameters, these values have to be corrected to achieve electric power, electric efficiency and exhaust gas temperature after diffuser, close to real (given by the producer). To facilitate correction of these parameters, the following relative errors (deviations) are given:

– electric power

$$\delta_{Nel} = \frac{N_{el}^{comp} - N_{el}^{real}}{N_{el}^{real}}$$

– electric efficiency

$$\delta_{\eta el} = \frac{\eta_{el}^{comp} - \eta_{el}^{real}}{\eta_{el}^{real}}$$

– combustion gas temperature after the diffuser

$$\delta_{Tg3} = \frac{T_{g3}^{comp} - T_{g3}^{real}}{T_{g3}^{real}}$$

Furthermore, the fuel mass flow rate relative error is calculated, which is used to determine the mean temperature of the combustion, and so its correction should be done in the first instance. Because the flame has different temperatures depending on its zone we operate on the mean combustion temperature. Relative error of fuel mass flow rate:

$$\delta_{mpal} = \frac{\dot{m}}{\dot{m}}$$

The real fuel mass flow rate is calculated from transformed electric efficiency formula:

$$\dot{m} \qquad \frac{N_{el}^{real*}}{\eta_{el}^{al*} \cdot LHV}$$

That mass flow rate is calculated with an assumption, that gas turbine unit is fuelled by pure methane, which is the main component of natural gas, hence star in the index. As a fuel, in all calculations, to simplify it is taken high-methane natural gas reduced only to methane.

Pressure losses after the turbine side have been defined as a difference between atmospheric pressure and pressure in the end point of combustion gas expansion, and they consist of losses on the diffuser and losses on HRSG:

$$\Delta p_{\omega} = p_{atm} - p_{g2} = \Delta p_{DIFF} + \Delta p_{HRSG}$$

In the first stage of simulation we endeavour to reach satisfactory convergence with catalogue parameters (el. power, el. efficiency, temperature after diffuser), which are given for gas turbine unit operating in simple cycle, therefore HRSG pressure losses shouldn't be taken into consideration. Absolute deviation values of all considered comparative parameters should be as small as possible and close to others. After correction of input parameters and fixing their values, we can set backpressure caused by HRSG, what will result in the gas turbine unit power and efficiency decrease, as well as exhaust gas temperature increase.

Instead of isentropic efficiency of the compressor and turbine, it was decided to take their polytropic efficiencies, which unlike isentropic can be considered con-

stant, not changing with change of pressure ratio. Owing to this, the model can be utilized to simulate gas turbine unit in different pressure ratio values and in search of the optimal pressure ratio.

3. MODEL OF WORKING MEDIA

Such characteristic parameters for air, combustion gas, or fuels, as specific heat capacity or specific enthalpy (hereinafter abbreviated as heat capacity and enthalpy, respectively) are functions of temperature, as well as of pressure. For order of magnitude of pressure values that occur in a gas turbine, it has been recognized that for gas turbine modelling purposes in order to simulate its operation in combined cycle, use of semi perfect gas model will be sufficient. From among available thermodynamic properties models of species participating in processes occurring in gas turbine, that is models of air, fuel and combustion gas constituents, NASA [Moran, Shapiro 2006] model has been chosen.

In the current model, total enthalpy of substance is a sum of standard enthalpy of formation and physical enthalpy:

$$\bar{h}(T) \equiv \bar{h}_f^o + \left[\bar{h}(T) - \bar{h}\left(T_{ref}\right)\right] = \bar{h}_f^o + \Delta\bar{h}$$

For elements, total enthalpy is equal to physical enthalpy. As a reference state, a temperature of 298,15 K is established. In that temperature total enthalpy of element equals zero, while total enthalpy of compound is equal to the standard enthalpy of formation.

Non-dimensional forms of heat capacity and total enthalpy present in the form of nonlinear equations are as follows:

$$\frac{\bar{c}_p(T)}{\tilde{\imath}} \qquad \neg^{-2} + a_2 \cdot T^{-1} + a_3 + a_4 \cdot T + a_5 \cdot T^2 + a_6 \cdot T^3 + a_7 \cdot T^4$$

$$\frac{\bar{h}(T)}{\tilde{\imath}} \qquad \cdot T^{-2} + a_2 \cdot \frac{\ln T}{T} + a_3 + a_4 \cdot \frac{T}{2} + a_5 \cdot \frac{T^2}{3} + a_6 \cdot \frac{T^3}{4} + a_7 \cdot \frac{T^4}{5} + \frac{a_8}{T}$$

Enthalpy equation was obtained by the integration of $c_p(T)$ respectively to T. Coefficient a8 is an integral constant. Values of coefficients a1÷a8, among others for dry air, water, nitrogen, oxygen, argon and carbon dioxide, methane, and also for many other substances, are presented in the above mentioned document. There are different values of this coefficients regarding temperature range: from 200 K to 1000 K, and 1000 K to 6000 K. To obtain heat capacity and enthalpy formulae expressed in kJ/(kgK) and kJ/kg, respectively, it is necessary to replace the universal gas constant with the individual gas constant.

4. AIR COMPOSITION

Determination of air composition flowing through a compressor, at given environment conditions, is the starting point for compressor calculations. Gas turbine producers usually show their nominal parameters at ISO conditions, thus, when finding gas turbine design point, calculations are to be led for that data. Composition of dry air can be acknowledged as constant. Molar fraction of moisture in the air, according to Dalton's law, is directly proportional to the partial pressure of water contained in the air, which directly depends on its relative humidity and on the saturation pressure of water:

$$p_{H2O}^{air} = \phi \cdot p_{sat}\left(T_{atm}\right)$$

Saturation pressure of water is, in turn, a function of its temperature. Saturation pressure of water in the atmospheric air is then a function of ambient air temperature. For perfect and semi perfect gases, molar fractions of the individual components in the mixture are equal to their volume fractions. Treating the air as the semi perfect gas, the volume fraction of moisture in the air is:

$$x_{H2O}^{air} = \frac{p_{H2O}^{air}}{p_{atm}}$$

In the ISO conditions the volume fraction of moisture in the air is ca. 1,01%.

The volume fraction of dry air in humid air:

$$x_{dry\ air}^{air} = 1 - x_{H2O}^{air}$$

The next steps are intended to define functions of the heat capacity and enthalpy of air, which we will operate in the calculation of the compressor.

Molar mass of humid air (hereinafter abbreviated as air):

$$M_{air} = M_{dry\ air} \cdot x_{dry\ air}^{air} + M_{H2O} \cdot x_{H2O}^{air}$$

Molar mass of dry air and water, and other necessary in further calculations physical constants can be found in [Moran, Shapiro 2006].

Gas constant for the air:

$$R_{air} = \frac{\tilde{\imath}}{M_{air}}$$

The equations of heat capacity and enthalpy of air, after defining the vector of coefficients of air:

$$\mathbf{P1} = \mathbf{A1} \cdot x_{dry\ air}^{air} + \mathbf{B1} \cdot x_{H2O}^{air}$$

Will take the forms:

$$c_{p,\,air}\left(T\right) = R_{air} \cdot \left(PI_1 \cdot T^{-2} + PI_2 \cdot T^{-1} + PI_3 + PI_4 \cdot T + PI_5 \cdot T^2 + PI_6 \cdot T^3 + PI_7 \cdot T^4 \right)$$

$$h_{air}\left(T\right) = R_{air} \cdot \left(-PI_1 \cdot T^{-1} + PI_2 \cdot \ln T + PI_3 \cdot T + PI_4 \cdot \frac{T^2}{2} + PI_5 \cdot \frac{T^3}{3} + PI_6 \cdot \frac{T^4}{4} + PI_7 \cdot \frac{T^5}{5} + PI_8 \right)$$

Figure 1 after the letter representing the vector of coefficients says that this vector is applicable in the first range of temperature (up to 1000 K). Pressure ratios found in gas turbine compressors are not big enough to raise the air temperature to exceed 1000 K, so it is sufficient to create a vector of coefficients of air only for the first temperature range.

5. AIR FILTER

Atmospheric air, sucked from the environment, is routed to the compressor by the channel through the inlet and filter. The pressure behind the air filter:

$$p_{a1} = p_{atm} - \Delta p_{AF}$$

Pressure drop along the filter is usually in the range of 5÷15 mbar. It is considered that the air temperature along the filter is constant, and thus

$$T_{a1} = T_{atm}$$

6. COMPRESSOR

The air in the compressor undergoes compression, through which increases its temperature. From there, the air reaches the combustion chamber. General scheme of the calculation of the compressor with air filter is shown in Fig. 3.

The main aim of the compressor calculation is to determine the enthalpy increment of compressed air and its final temperature. This can be done starting from the definition of isentropic compression efficiency:

$$\eta_{s,\,compression} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\eta_{s,\,compression} \equiv \eta_{s,\,COM}$$

To determine the increment of air enthalpy in an isentropic compression, its temperature in the hypothetical end point of an isentropic compression has to be found firstly (air temperature before the compressor is known). This was done numerically, by solving the entropy change equation [Walsh, Fletcher 2004]:

$$s_2 - s_1 = \int_{T1}^{T2} \frac{c_p(T)}{T} dT - R \cdot \ln \frac{p_2}{p_1}$$

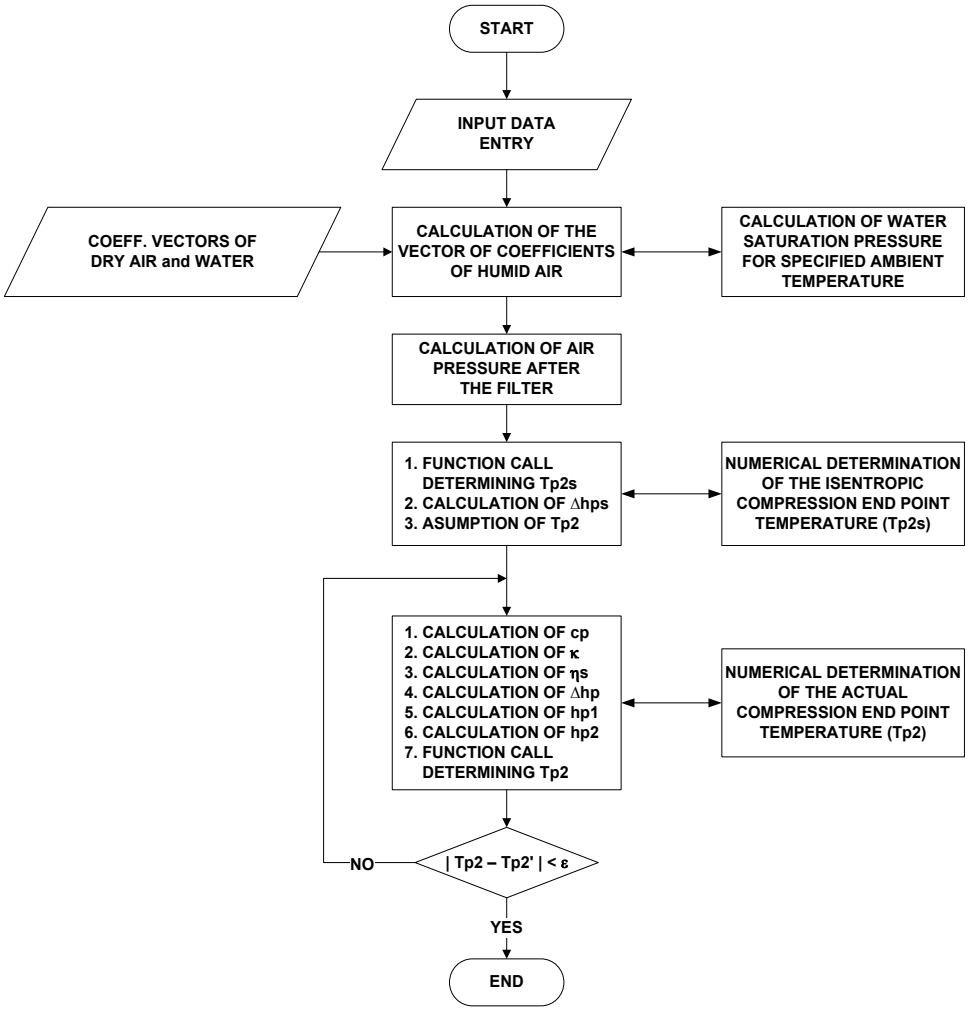

Fig. 3. Flowchart of a compressor and an air filter calculation algorithm

Which to the case is as follows:

$$s_{a2s} - s_{a1} = \int_{Ta1}^{Ta2s} \frac{c_{p,air}(T)}{T} dT - R_{air} \cdot \ln \frac{p_{a2}}{p_{a1}}$$

When compressing is ideal (reversible), there is no increase in entropy, so

$$s_{a2s} = s_{a1}$$

Compressor pressure ratio:

$$\pi = \frac{p_{a2}}{p_{a1}}$$

Given these two relationships, equation takes the form:

$$\int_{Ta1}^{Ta2s} \frac{c_{p,air}(T)}{T} dT = R_{air} \cdot \ln \pi$$

This equation, after integration $c_p(T)/T$, is placed in the procedure that returns T_{a2s}. The isentropic air enthalpy increment can now be calculated:

$$\Delta h_{air,s} = h_{a2s} - h_{a1} = h_{air}\left(T_{a2s}\right) - h_{air}\left(T_{a1}\right)$$

First, non-iterated part of the compressor calculations ends on the assumption of air temperature after the compressor, T_{a2}.

The next unknown, that has to be calculated, is the compressor isentropic efficiency, given by [Devernay 2007]:

$$\eta_{s,COM} = \frac{\pi^{\frac{\kappa_{air}-1}{\kappa_{air}}} - 1}{\pi^{\frac{\kappa_{air}-1}{\kappa_{air} \cdot \eta_{p,COM}}} - 1}$$

Where the isentropic exponent of air compression:

$$\kappa_{air} = \frac{1}{1 - \frac{R_{air}}{c_{p,air}\Big|_{Ta1}^{Ta2}}}$$

The mean heat capacity in the temperature range from T_1 to T_2 defines the relationship

$$c_p\Big|_{T1}^{T2} = \frac{\Delta h}{\Delta T}$$

Taking in this case the form:

$$c_{p,air}\Big|_{Ta1}^{Ta2} = \frac{h_{air}\left(T_{a2}\right) - h_{air}\left(T_{a1}\right)}{T_{a2} - T_{a1}}$$

The second part of the compressor calculations begins from the mean heat capacity calculation in the temperature range T_{a1} and T_{a2}, would then be able to determine the isentropic exponent and the isentropic efficiency.

Knowing the compressor isentropic efficiency, we are able to determine the actual increase in enthalpy of the air in the compressor:

$$\Delta h_{air} = \frac{\Delta h_{air,s}}{\eta_{s,COM}}$$

and in consequence, the enthalpy of the air at the end point of the actual compression, needed to determine the air temperature after the compressor:

$$h_{a2} = h_{air}\left(T_{a1}\right) + \Delta h_{air}$$

Numerically solving the equation

$$h_{a2} = h_{air}\left(T_{a2}\right)$$

We find T_{a2}. However, this is not the actual air temperature after the compressor, because the mean value of heat capacity was determined based on the assumed temperature value T_{a2}. It is necessary to loop the second part of the compressor calculation, which in each iteration is updated the final compression temperature. At the end of each loop, the convergence condition is checked:

$$\left|T_{a2} - T'_{a2}\right| < \varepsilon$$

ε is set on 0,01 K.

7. COMBUSTION CHAMBER

Compressed air brought to the combustor mixes with the fuel, and is followed by combustion – i.e. exothermic reaction between the fuel and oxidant. The released energy causes the heating up of the gases.

Starting the calculations of the combustion chamber, molar compositions of air and fuel (methane) feed flows must be previously defined. For the methane model, its coefficients are contained the **Z1** vector.

It was assumed the following molar composition of dry air:

$$x_{N2}^{dry\ air} = 78,082\ \%$$

$$x_{O2}^{dry\ air} = 20,946\ \%$$

$$x_{Ar}^{dry\ air} = 0,934\ \%$$

$$x_{CO2}^{dry\ air} = 0,038\ \%$$

Molar composition of air, for i = N_2, O_2, Ar, CO_2:

$$x_i^{air} = x_i^{dry\ air} \cdot x_{dry\ air}^{air}$$

Recall that the molar fraction of water in the air is already known. Molar mass of air with the above-mentioned composition, for i = N_2, O_2, Ar, CO_2, H_2O:

$$M_{air} = \sum_i M_i \cdot x_i^{air}$$

As the air and methane flowing into the combustion chamber have a temperature below 1000 K, while the combustion occurs at a temperature above 1000 K, there is a need to create vectors of coefficients for models of air and combustion gas components for the two temperature ranges (up to 1000 K and from 1000 K), and for the methane model for the first range. Missing vectors: **B2**, **C1**, **C2**, **D1**, **D2**, **E1**, **E2**, **F1**, **F2** and **Z1** are to be used in the molar enthalpy equation, thus obtaining the molar enthalpy formulas for H_2O, N_2, O_2, Ar and CO_2, respectively, for both temperature ranges, and CH_4 for the first range.

Complete combustion of methane in an excess of air is given by:

$$CH_4 + 2\bar{\lambda}\cdot\left(\frac{x_{N2}^{air}}{x_{O2}^{air}}N_2 + O_2 + \frac{x_{Ar}^{air}}{x_{O2}^{air}}Ar + \frac{x_{CO2}^{air}}{x_{O2}^{air}}CO_2 + \frac{x_{H2O}^{air}}{x_{O2}^{air}}H_2O\right)$$

$$\rightarrow 2\bar{\lambda}\cdot\frac{x_{N2}^{air}}{x_{O2}^{air}}N_2 + d\,O_2 + 2\bar{\lambda}\cdot\frac{x_{Ar}^{air}}{x_{O2}^{air}}Ar + \left(f + 2\bar{\lambda}\cdot\frac{x_{CO2}^{air}}{x_{O2}^{air}}\right)CO_2 + \left(b + 2\bar{\lambda}\cdot\frac{x_{H2O}^{air}}{x_{O2}^{air}}\right)H_2O$$

After balancing atoms of individual elements we find d, f and b:

$$CH_4 + 2\bar{\lambda}\cdot\left(\frac{x_{N2}^{air}}{x_{O2}^{air}}N_2 + O_2 + \frac{x_{Ar}^{air}}{x_{O2}^{air}}Ar + \frac{x_{CO2}^{air}}{x_{O2}^{air}}CO_2 + \frac{x_{H2O}^{air}}{x_{O2}^{air}}H_2O\right)$$

$$\rightarrow 2\bar{\lambda}\cdot\frac{x_{N2}^{air}}{x_{O2}^{air}}N_2 + 2\left(\bar{\lambda}-1\right)O_2 + 2\bar{\lambda}\cdot\frac{x_{Ar}^{air}}{x_{O2}^{air}}Ar + \left(1 + 2\bar{\lambda}\cdot\frac{x_{CO2}^{air}}{x_{O2}^{air}}\right)CO_2 + \left(2 + 2\bar{\lambda}\cdot\frac{x_{H2O}^{air}}{x_{O2}^{air}}\right)H_2O$$

where

$$\bar{\lambda} = \frac{i}{i}$$

On the basis of the first principle of thermodynamics for reactive systems we create energy balance of the combustion chamber:

$$\eta_{CC}\cdot\left[1\cdot\bar{h}I_{CH4}\left(T_{fuel}\right) + 2\bar{\lambda}\cdot\left(\begin{array}{l}\frac{x_{N2}^{air}}{x_{O2}^{air}}\cdot\bar{h}I_{N2}\left(T_{a2}\right) + 1\cdot\bar{h}I_{O2}\left(T_{a2}\right) + \frac{x_{Ar}^{air}}{x_{O2}^{air}}\cdot\bar{h}_{Ar}\left(T_{a2}\right) + \frac{x_{CO2}^{air}}{x_{O2}^{air}}\cdot\bar{h}_{CO2}\left(T_{a2}\right)\\ + \frac{x_{H2O}^{air}}{x_{O2}^{air}}\cdot\bar{h}_{H2O}\left(T_{a2}\right)\end{array}\right)\right] =$$

$$= 2\bar{\lambda}\cdot\frac{x_{N2}^{air}}{x_{O2}^{air}}\cdot\bar{h}2_{N2}\left(T_{g0}\right) + 2\left(\bar{\lambda}-1\right)\cdot\bar{h}2_{O2}\left(T_{g0}\right) + 2\bar{\lambda}\cdot\frac{x_{Ar}^{air}}{x_{O2}^{air}}\cdot\bar{h}2_{Ar}\left(T_{g0}\right) + \left(1 + 2\bar{\lambda}\cdot\frac{x_{CO2}^{air}}{x_{O2}^{air}}\right)\cdot\bar{h}2_{CO2}\left(T_{g0}\right)$$

$$+\left(2 + 2\bar{\lambda}\cdot\frac{x_{H2O}^{air}}{x_{O2}^{air}}\right)\cdot\bar{h}2_{H2O}\left(T_{g0}\right)$$

The excess air coefficient λ is found numerically from the above equation. If $\eta_{CC} = 1$, the mean temperature of combustion equals the mean adiabatic flame temperature.

Once the solution has been computed, the mass fuel-to-air ratio can be used to obtain the mass flow rate of fuel and compressed air. Mass fuel-to-air ratio is defined by:

$$\alpha = \frac{\dot{m}_{fuel}}{\dot{m}_{air}}$$

In this case:

$$\alpha = \frac{1 \cdot M_{CH4}}{2\bar{\lambda} \cdot \left(\frac{x_{N2}^{air}}{x_{O2}^{air}} + 1 + \frac{x_{Ar}^{air}}{x_{O2}^{air}} + \frac{x_{CO2}^{air}}{x_{O2}^{air}} + \frac{x_{H2O}^{air}}{x_{O2}^{air}} \right) \cdot M_{air}}$$

Having the relative fuel mass flow rate, the mass flow rate of fuel and the mass flow rate of air flowing through the compressor can be determined:

$$\dot{m}_{fuel} = \dot{m}_{air} \alpha$$

$$\dot{m}_{air} = \frac{\dot{m}_{fuel}}{\alpha}$$

To determine the composition of the combustion gas, the molar number has to be determined previously:

$$N_{gas} = 2\bar{\lambda} \cdot \frac{x_{N2}^{air}}{x_{O2}^{air}} + 2\left(\bar{\lambda} - 1\right) + 2\bar{\lambda} \cdot \frac{x_{Ar}^{air}}{x_{O2}^{air}} + \left(1 + 2\bar{\lambda} \cdot \frac{x_{CO2}^{air}}{x_{O2}^{air}}\right) + \left(2 + 2\bar{\lambda} \cdot \frac{x_{H2O}^{air}}{x_{O2}^{air}}\right)$$

Molar volumetric composition of combustion gas:

$$x_{N2}^{gas} = \frac{2\bar{\lambda} \cdot \frac{x_{N2}^{air}}{x_{O2}^{air}}}{N_{gas}}$$

$$x_{O2}^{gas} = \frac{2\left(\bar{\lambda} - 1\right)}{N_{gas}}$$

$$x_{Ar}^{gas} = \frac{2\bar{\lambda} \cdot \frac{x_{Ar}^{air}}{x_{O2}^{air}}}{N_{gas}}$$

$$x_{CO2}^{gas} = \frac{1 + 2\bar{\lambda} \cdot \frac{x_{CO2}^{air}}{x_{O2}^{air}}}{N_{gas}}$$

$$x_{H2O}^{gas} = 1 - \left(x_{N2}^{gas} + x_{O2}^{gas} + x_{Ar}^{gas} + x_{CO2}^{gas}\right)$$

Combustion gas coefficient vectors:

$$\mathbf{S1} = \mathbf{C1} \cdot x_{N2}^{gas} + \mathbf{D1} \cdot x_{O2}^{gas} + \mathbf{E1} \cdot x_{Ar}^{gas} + \mathbf{F1} \cdot x_{CO2}^{gas} + \mathbf{B1} \cdot x_{H2O}^{gas}$$

$$\mathbf{S2} = \mathbf{C2} \cdot x_{N2}^{gas} + \mathbf{D2} \cdot x_{O2}^{gas} + \mathbf{E2} \cdot x_{Ar}^{gas} + \mathbf{F2} \cdot x_{CO2}^{gas} + \mathbf{B2} \cdot x_{H2O}^{gas}$$

The equations of heat capacity and enthalpy of combustion gas, after having calculated the gas constant (i = N_2, O_2, Ar, CO_2, H_2O)

$$R_{gas} = \frac{\tilde{\imath}}{M_{gas}} - \frac{\tilde{}}{\sum_i M_i \cdot x_i^{gas}}$$

will take the forms:

$$c_{p,\,gas}^{\,I}(T) = R_{gas} \cdot \left(SI_1 \cdot T^{-2} + SI_2 \cdot T^{-1} + SI_3 + SI_4 \cdot T + SI_5 \cdot T^2 + SI_6 \cdot T^3 + SI_7 \cdot T^4 \right)$$

$$c_{p,\,gas}^{\,II}(T) = R_{gas} \cdot \left(S2_1 \cdot T^{-2} + S2_2 \cdot T^{-1} + S2_3 + S2_4 \cdot T + S2_5 \cdot T^2 + S2_6 \cdot T^3 + S2_7 \cdot T^4 \right)$$

$$h_{gas}^{\,I}(T) = R_{gas} \cdot \left(-SI_1 \cdot T^{-1} + SI_2 \cdot \ln T + SI_3 \cdot T + SI_4 \cdot \frac{T^2}{2} + SI_5 \cdot \frac{T^3}{3} + SI_6 \cdot \frac{T^4}{4} + SI_7 \cdot \frac{T^5}{5} + SI_8 \right)$$

$$h_{gas}^{\,II}(T) = R_{gas} \cdot \left(-S2_1 \cdot T^{-1} + S2_2 \cdot \ln T + S2_3 \cdot T + S2_4 \cdot \frac{T^2}{2} + S2_5 \cdot \frac{T^3}{3} + S2_6 \cdot \frac{T^4}{4} + S2_7 \cdot \frac{T^5}{5} + S2_8 \right)$$

8. TURBINE

Combustion gas, flowing in to the turbine expands, which decreases its temperature, so the process is the reverse to that which occurs in the compressor. The turbine calculation algorithm is analogous to the algorithm of the compressor calculation. We strive to determine the decrease in combustion gas enthalpy, and its final temperature.

For expansion process, isentropic efficiency is defined as:

$$\eta_{s,\,expansion} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

$$\eta_{s,\,expansion} \equiv \eta_{s,\,TUR}$$

To determine the decrease of combustion gas enthalpy during isentropic expansion, the temperature at the hypothetical end point of the isentropic expansion must be found first (combustion gas temperature before the turbine is known). As with the compressor calculations, the entropy change equation was used. The combustion gas temperature before the turbine, in practice is always greater than 1000 K, and the temperature after the turbine is less than 1000 K. Using NASA media model [6] in the turbine calculations, two temperature ranges must be therefore taken into account: T_{g2} (T_{g2s})÷1000 [K] and 1000÷T_{g1}[K]. This time the entropy change equation looks as follows:

$$s_{g2s} - s_{g1} = -\int_{Tg2s}^{1000K} \frac{c_{p,gas}^{I}(T)}{T} dT - \int_{1000K}^{Ts1} \frac{c_{p,gas}^{II}(T)}{T} dT - R_{gas} \cdot \ln \frac{p_{g2}}{p_{g1}}$$

The negative sign before the integrals derive from the fact that the ranges of integration were reversed. When there is ideal expansion, there is no entropy increase, so

$$s_{g2s} = s_{g1}$$

Taking this into account, the equations takes the form:

$$-\int_{Tg2s}^{1000K} \frac{c_{p,gas}^{I}(T)}{T} dT - \int_{1000K}^{Tg1} \frac{c_{p,gas}^{II}(T)}{T} dT = R_{gas} \cdot \ln \frac{p_{g2}}{p_{g1}}$$

The combustion gas temperature before the turbine:

$$T_{g1} = T_{g0} - \Delta T_{CC}$$

The combustion gas pressure before the turbine:

$$p_{g1} = p_{g2} \cdot (1 - \xi_{CC}) = \pi \cdot p_{g1} \cdot (1 - \xi_{CC})$$

The combustion gas pressure after the turbine:

$$p_{g2} = p_{atm} + \Delta p_{\omega}$$

Equation after integration $c_p(T)/T$ for two temperature ranges, and placing in the procedure, returned T_{g2s}. With this temperature, we can calculate the isentropic enthalpy decrease of combustion gas during isentropic expansion.

$$\Delta h_{gas,s} = h_{gas}^{II}(T_{g1}) - h_{gas}^{I}(T_{g2s})$$

Before proceeding to the second, iterated part of the turbine calculation, the combustion gas temperature at the end point of actual expansion (T_{g2}) must be assumed.

Next, we aim to calculate isentropic efficiency of turbine [Devernay 2007]:

$$\eta_{s,TUR} = \frac{1 - \left(p_{g2} / p_{g1} \right)^{\frac{(\kappa_{gas}-1)\cdot\eta_{p,TUR}}{\kappa_{gas}}}}{1 - \left(p_{g2} / p_{g1} \right)^{\frac{\kappa_{gas}-1}{\kappa_{gas}}}}$$

where the isentropic exponent of combustion gas expansion:

$$\kappa_{gas} = \frac{1}{1 - \frac{R_{gas}}{c_{p,gas}\big|_{Tg2}^{Tg1}}}$$

Using the formula for the mean heat capacity, we arrive at the following form:

$$c_{p,gas}\big|_{Tg1}^{Tg2} = \frac{h_{gas}^{II}(T_{g1}) - h_{gas}^{I}(T_{g2})}{T_{g1} - T_{g2}}$$

Determination of the mean heat capacity of expanding combustion gas constitutes the beginning of the second part of the turbine calculations. The next steps are

$$w_{COM} = -\frac{\Delta h_{air}}{\eta_{mech}}$$

the determination of the isentropic exponent and turbine isentropic efficiency.

Knowing the turbine isentropic efficiency, we can find the actual enthalpy decrease of the combustion gas:

$$\Delta h_{gas} = \eta_{s,TUR} \cdot \Delta h_{gas,s}$$

and by that the enthalpy of the air at the end point of an actual expansion (irreversible):

$$h_{g2} = h_{gas}^{II}\left(T_{g1}\right) - \Delta h_{gas}$$

Solving the equation numerically

$$h_{g2} = h_{gas}^{I}\left(T_{g2}\right)$$

we calculate T_{g2}. Since mean heat capacity was calculated for the assumed combustion gas temperature T_{g2}, we must return to the calculation of c_p, providing the present value of the final expansion temperature, leading the calculation of the loop until the condition of convergence is satisfied:

$$\left|T_{g2} - T'_{g2}\right| < \varepsilon$$

ε is fixed to 0,01 K.

The combustion gas temperature after the diffuser:

$$T_{g3} = T_{g2} - \Delta T_{DIFF}$$

9. ELECTRIC EFFICIENCY AND POWER OF A GAS TURBINE UNIT

As it is not needed to know the absolute mass flow rates to find electric efficiency of the unit, we can use the following form:

$$\eta_{el} = \eta_{GEN}\left[\frac{w_{COM} + (1+\alpha)w_{TUR}}{\alpha LHV}\right]$$

where:

Power of a gas turbine unit:

$$N_{el} = \eta_{GEN}\left(N_{COM} + N_{TUR}\right)$$

where:

$$w_{TUR} = \eta_{mach}\Delta h_{gas}$$

$$N_{COM} = \dot{m}_{air} w_{COM}$$
$$N_{TUR} = \dot{m}_{gas} w_{TUR}$$

10. SUMMARY

The mathematical model of a gas turbine presented in this paper, after implementation into the computer program, allows for fast simulation of gas turbine operation in nominal conditions. Conducted simulations have proved that the determination of input parameters, so that the divergences of considered parameters were less than one percent against the real values, took only a while. By using numerical methods, nonlinear character of concerned processes occurring in the system was preserved, thereby achieving realistic and precise results. The simulator can be useful as a tool to assist in the initial design stage of a combined cycle, during the research studies, or during the analysis of an existing system. Furthermore, it has cognitive value. Using the tool, one should be aware of the limitations and simplifications of the model used. Knowing the data of a flue gas after the turbine, we can proceed to steam cycle calculations. Further extended, the model can serve as a simulator of a gas turbine operation in off-design conditions.

NOMENCLATURE

A1	dry air model coefficient vector
B1, B2	water model coefficient vectors
C1, C2	nitrogen model coefficient vectors
D1, D2	oxygen model coefficient vectors
E1, E2	argon model coefficient vectors
F1, F2	carbon dioxide model coefficient vectors
P1	air model coefficient vector
S1, S2	combustion gas model coefficient vectors
Z1	methane model coefficient vector
$c_p, \overline{c}_p$	specific heat capacity, isobaric [kJ/(kg*K)], molar specific heat capacity, isobaric [kJ/(kmol*K)]
$h, \overline{h}$	specific enthalpy [kJ/kg], molar specific enthalpy [kJ/kmol]
LHV	lower heating value [kJ/kg]
$\dot{m}$	mass flow rate [kg/s]
M	molar mass [kg/kmol]
$\dot{n}$	molar flow rate [kmol/s]
N	molar number [kmol]; power on the shaft [kW]
p	pressure [bar, kPa]
R	gas constant [kJ/(kg*K)]
$\tilde{R}$	universal gas constant, 8,31451 [kg/(kmol*K)]

s	specific entropy [kJ/(kg * K)]
T	temperature [°C, K]
w	specific work [kJ/kg]
x	molar/volumetric fraction [%]

Greek letters

α	relative mass stream of fuel [-]
δ	relative error [%]
ε	absolute convergence of temperature [°C, K]
ϕ	relative humidity of air [%]
κ	isentropic exponent [-]
η	efficiency [%]
λ	air excess coefficient [-]
ω	turbine side (at index)

List of index abbreviations

I, II	temperature ranges: 0÷1000 [K], 1000÷6000 [K], respectively
AF	air filter
atm	atmospheric
CC	combustion chamber
COM	compressor
comp	computational
DIFF	diffuser
el	electric
gas	combustion gas
GEN	electric generator
HRSG	heat recovery steam generator
mech	mechanical
p	polytropic
s	isentropic
sat	saturated
stoich	stoichiometric
TUR	turbine

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METODA MODELOWANIA SPRAWNOŚCI TURBINY GAZOWEJ

Streszczenie

Zwiększający się w światowym bilansie produkcji energii elektrycznej udział energii wytwarzanej z gazu ziemnego warunkuje tempo przyrostu ilości elektrowni i elektrociepłowni gazowo parowych. Pomimo wysokich cen gazu ziemnego, bloki gazowo parowe są atrakcyjne dla inwestorów z uwagi na: najwyższą sprawność wśród stosowanych na szeroką skalę instalacji energetycznych zasilanych paliwami kopalnymi, dopracowaną technologię skutkującą wysoką niezawodnością, wysoką elastyczność eksploatacyjną, słabe oddziaływanie na środowisko (niskie emisje dzięki wysokiej sprawności i opalaniu gazem ziemnym). Podstawą obliczeń układu kombinowanego na etapie koncepcjonowania/projektowania wstępnego lub podczas jego analizowania są obliczenia turbozespołu gazowego pracującego w punkcie konstrukcyjnym. W artykule przedstawiono krok po kroku proces modelowania matematycznego zespołu turbiny gazowej pod kątem implementacji modelu w programie obliczeniowym. Założono stacjonarną pracę układu przy pełnym obciążeniu. Przewidywanym zastosowaniem modelu jest przede wszystkim symulacja turbiny gazowej skojarzonej z parowym kotłem odzyskowym, tym samym działającej w kombibloku. Omówiono problemy związane z poruszanym zagadnieniem oraz zaproponowano sposoby ich rozwiązania.